



**US Army Corps
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Robust Prediction of Hydraulic Roughness

by S. Kyle McKay¹ and J. Craig Fischenich²

OVERVIEW: The National River Restoration Science Synthesis (NRRSS) demonstrated that, in 2007, river and stream restoration projects and funding were at an all time high and increasing exponentially (Bernhardt et al. 2007). Increasingly, these restoration projects rely on “soft” engineering techniques involving planting riparian vegetation to alter channel and floodplain hydraulics or geomorphology. The ability to quantify the influence of vegetation on channel and floodplain hydraulics, in particular hydraulic roughness, is critical for flood control concerns; however, diversity of vegetation type and behavior makes this parameter very difficult to quantify repeatedly and accurately.

A fundamental concept of hydraulic theory in the context of river engineering is the influence of boundary conditions on flow through natural environments. This technical note presents a tool for estimating hydraulic roughness from boundary conditions in rivers. Hydraulic roughness, or resistance, is herein defined as the primary factor influencing retarding or resisting forces exerted by channel boundaries on stream flow. Calculation of hydraulic resistance is not a trivial matter due to the multitude of factors influencing roughness (e.g., bed material, bed forms, cross-sectional and planform variability, vegetation, etc.). This document will present a theory of hydraulic resistance estimation, a synthesis of many resistance estimation techniques into a spreadsheet model, and an application of said model to Ham Branch, a tributary of the Trinity River. The diversity of methods applied in the model will allow users to isolate effects of diverse contributions to roughness (e.g., grain v. vegetative) and create a weight-of-evidence for an estimation of hydraulic roughness.

HYDRAULIC ROUGHNESS: THEORY: The laws of conservation of energy and momentum must account for hydraulic resistive forces in calculation of open channel hydraulics. Uniform flow conditions require driving and resisting forces to be balanced; that is, flow is not accelerating or decelerating, so average channel cross-section, slope, and velocity are assumed to be constant under constant discharge conditions. In natural streams velocity or discharge must often be estimated or calculated using other flow parameters, most commonly hydraulic radius, energy slope (or some approximation), and some estimate of channel roughness. Even though natural streams do not strictly comply with uniform flow assumptions, uniform flow conditions are often assumed to simplify velocity and discharge computations. Average velocity in river engineering applications is commonly calculated using one of three equations: Manning, Chezy, or Darcy-Weisbach (Yen 2002; equations shown below):

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$$U = \frac{k_n R^{1/6}}{n} \sqrt{RS_f} \text{ (Manning)}$$

$$U = C_z \sqrt{RS_f} \text{ (Chezy)}$$

$$U = \sqrt{\frac{8g}{f}} \sqrt{RS_f} \text{ (Darcy-Weisbach)}$$

where U is cross-section averaged velocity, R is hydraulic radius, S_f is friction or energy slope, k_n is a unit correction factor ($k_n = 1.0 \text{ m}^{1/3}/\text{s} = 1.486 \text{ ft}^{1/3}/\text{s}$), n is Manning's coefficient, C_z is Chezy's coefficient, f is Darcy-Weisbach coefficient, and g is gravitational acceleration.

Rigorous application of these equations requires knowledge of S_f , which require extensive data collection and analysis. However, because uniform flow conditions are generally assumed (i.e., channel cross-section and velocity are relatively similar or begin and end with similar values throughout the longitudinal domain), friction slope, water surface slope, and bed slope are assumed to be equivalent. Therefore, for uniform or near-uniform flow, friction slope can be assumed to be equivalent to bed slope ($S_f \approx S_b$), though water surface slope is most commonly substituted for S_f ($S_f \approx S_w$).

For each of these equations, resistance or roughness coefficients must be derived or assumed, often from previous empirical work. Darcy-Weisbach coefficient is dimensionless and theoretical; Chezy and Manning coefficients were derived empirically. Chezy and Manning's equations are by far the most common resistance relations used among practitioners, with Manning's equation leading amongst river engineers. References to and application of roughness coefficients in this document refer primarily to Manning's n .

Empirically derived resistance relations like Manning's equation reliably account for total frictional losses in natural channels, though there are inherent sources of error. Sources of resistance in rivers may include but are not limited to: boundary surface roughness (from sediment or vegetation), form roughness due to bedforms or channel irregularities in cross-section, planform irregularities (e.g., meanders), flow obstructions (e.g., debris jams), and other flow properties (e.g., stage, discharge, turbulence, sediment load and viscosity). Methods exist to account for each of these contributions separately; however, accurately distributing roughness to these elements is very difficult (Chow 1959).

Estimation techniques generally account for one of four categories or sources of roughness: total, grain, bedform, or vegetative. In addition to difficulties in estimating roughness even in a roughly homogeneous section of a channel, many rivers exhibit variable roughness conditions throughout the lateral domain (e.g., vegetated floodplain of a sand bed stream). In these complex channels a composite roughness must be calculated. The following sections outline different techniques and equations for estimating roughness for each roughness type; each technique is explained and developmental limitations are addressed.

TOTAL ROUGHNESS: There are generally two categories for estimating total roughness: those relying on extensive data collection and calibration (direct measurement) and those relying on field assessment and best professional judgment (analytical and handbook methods).

Direct Measurement. Direct measurement of flow resistance is a time consuming and often cumbersome process. Though important for model and prototype calibration and verification (e.g., from high water marks) this method is of little practical use for general prediction, so it is summarized briefly herein but is not discussed further. This method requires assembling appropriate instrumentation and physically measuring channel cross-section dimensions, flow depth or stage, and velocities at several locations in the cross-section. From these parameters discharge is determined, and with velocity and channel geometry, roughness can be calculated.

Flow depth and stage measurements can be made using non-recording gages which require manual readings (e.g., wading rod or crest-stage gage) or recording stage gages which provide a continuous record of water surface elevations with time (e.g., strip chart or digital recorders). Velocity-measuring devices range from simple mechanical devices, such as rotating-element and vertical-axis current meters, to highly sophisticated electrical systems, such as electromagnetic and acoustic meters. Many factors such as conditions under which measurements are made, availability of equipment and instruments needed, relative precision or accuracy requirements, and associated costs influence whether and which techniques should be applied. Equipment and precise methods for measurement of open channel flow are described in great detail in many USGS documents (e.g., National Handbook of Recommended Methods for Water Conservation 1977, Mueller and Wagner 2009) and in some detail in most introductory hydrology text books (e.g., Viessman and Lewis 2003, Chow et al. 1988). To arrive at a robust estimate of roughness, field measurement at multiple stages is recommended to identify flow depth dependencies of channel resistance. Alternatively, discharge- or stage-specific roughness estimates may be of greatest use for specific applications, such as bankfull discharge or particular flood return interval analyses.

Field Assessment. Field assessment methods refer to those that do not rely on direct measurement or detailed numerical analyses. Included in this category are photographic comparison, table estimates, and Cowan's approach. All approaches rely on best professional judgment, that is, the ability of users to select a reference channel that is similar to the reach they are evaluating and apply the appropriate roughness value.

Professional Judgment. The simplest (and least repeatable) estimates of hydraulic roughness are based on professional judgment from long-term field experience. An experienced river scientist or engineer estimates roughness for a channel by field observation. However, due to differences of experience or opinion, a team of experienced river scientists may examine the same reach and arrive at dramatically different estimates. This approach should only be applied when no other estimates or techniques are practicable.

Tables. Tables of Manning's n values published in Chow (1959) are arguably the most common source for selection of channel and floodplain roughness values. Chow provided minimum, normal, and maximum values of Manning's n for conduits, lined canals, and natural channels; however, of the 111 channels and floodplains presented in Chow's tables only 27 include vegetation.

Photographic Comparison. A number of authors have presented roughness prediction techniques where photographs have been taken of natural channels simultaneous to flow measurements. These calibrated photos and accompanying qualitative channel descriptions may then be applied to other reaches to estimate roughness. Table 1 provides a summary of these methods as well as notable features and limitations.

Table 1. Summary of Roughness Prediction Techniques Using Photographic Comparison.

Author (Year)	Features and Limitations
Chow (1959)	Photographs of 24 channels with qualitative channel descriptions and estimated Manning's n value 11 photographs show evidence of vegetation and only three appear to actually incorporate vegetation influence in total roughness
Barnes (1967)	Color photographs and descriptive data for 50 stream channels, many of which included vegetated banks Overbank flows omitted from calculations of n values
Aldridge and Garrett (1973)	Photographs of select Arizona channels and floodplains with accompanying descriptions of channel geometry and site conditions
Arcement and Schneider (1989)	Photographs for 15 densely vegetated floodplains Only known visual comparison method in which an attempt was explicitly made to identify roughness contribution due to vegetation Applied general procedure of Cowan (1956) and vegetation-density method proposed by Petryk and Bosmajian (1975) Measured vegetation density in floodplain and used an effective drag coefficient to calculate vegetation influence on total roughness Values for Manning's n ranged from 0.10 to 0.20 with vegetation contributions ranging from 0.065 to 0.145 (64-81 percent of total roughness)
Hicks and Mason (1991)	Comprehensive pictorial reference (78 New Zealand river reaches) Includes multiple photographs for each reach, bed material gradations, and a summary table with relevant hydraulic parameters Multiple discharges evaluated Avoided computation of flow resistance in floodplains; however, work provides insight into contribution of bank vegetation to roughness
Gillen (1996)	Color photographs and descriptive data for 10 streams in West Central Florida with vegetated banks
Coon (1998)	Color photographs and descriptive data for 21 streams in New York State with vegetated banks in growing and dormant stages
Phillips and Ingersoll (1998)	Color photographs and descriptive data for 14 rivers and canals in the arid southwestern United States (Arizona) Channels contained varying amounts of riparian vegetation Multiple discharges, bed forms, and vegetation types are addressed
Soong et al. (2008)	Color photographs and descriptive data for rivers and canals in Illinois. Database is maintained online and is under-development, but as of publication of this report, more than 40 channels have been cataloged

Cowan's Analytical Method. Cowan (1956) proposed a procedure for estimating Manning's n that accounts for contributions of various factors, including vegetation, to total flow resistance. The procedure assumes linearity, which implies that resistance of contributing factors can be summed to establish total resistance. Individuals using Cowan's approach use a table to select a base value for n , multiple adjustment factors, and degree of meandering.

$$n = (n_b + n_1 + n_2 + n_3 + n_4)m$$

where n_b is a base n value, n_1 is an addition for surface irregularities, n_2 is an addition for variation in channel cross-section, n_3 is an addition for obstructions, n_4 is an addition for vegetation, and m is a correction for meandering (see Cowan 1956 and Phillips and Tadayon 2007 for coefficient guidance).

Grain Roughness. In river engineering, the most commonly calculated constituent of channel roughness is that due to channel substrate, commonly referred to as grain or relative roughness. A number of analytical approaches exist to predict grain roughness from various channel properties. As previously explained, Manning's n is known to be flow depth dependent. As such, grain roughness predictors are divided into those that account for flow dependency and those that do not.

Flow Dependent Methods. The purpose of this paper is not to present a thorough review of these methods, but instead to present a few more commonly applied methods. Three methods applied in a number of studies (e.g., Marcus et al. 1992, Ghaffar et al. 2004, Conyers and Fonstad 2005) are those developed by Jarrett (1984), Limerinos (1970), and Bathurst (1985). Jarrett (1984) presents an empirical approach for estimating resistance in high-gradient mountain streams. His equation was derived from examination of dependencies in observed values and was calibrated and verified using data from the Rocky Mountains of Colorado. Limerinos (1970) developed an equation for predicting grain roughness as a function of logarithmic velocity distribution and relative roughness of channel substrate. This work recalibrated the previous work of Leopold and Wolman (1957) using 50 field measurements from 11 gravel-bed streams in northern coastal California. Bathurst (1985) developed a similar equation calibrated to 16 high-gradient British streams. Table 2 presents a summary of these techniques and their calibration ranges.

$$\begin{aligned} n &= 0.39S_f^{0.38}R^{-0.16} \quad \text{for } R = \text{feet} \\ n &= 0.32S_f^{0.38}R^{-0.16} \quad \text{for } R = \text{meters} \end{aligned} \qquad \text{Jarrett (1984)}$$

$$\frac{n}{R^{1/6}} = \frac{0.0926}{1.16 + 2.0 \log\left(\frac{R}{d_{84}}\right)} \quad \text{for } \text{Units} = \text{feet} \qquad \text{Limerinos (1970)}$$

$$\frac{n}{R^{1/6}} = \frac{0.1129}{1.16 + 2.0 \log\left(\frac{R}{d_{84}}\right)} \quad \text{for } \text{Units} = \text{meters}$$

$$\frac{n}{H^{1/6}} = \frac{0.2619}{5.62 \log\left(\frac{H}{d_{84}}\right) + 4.0} \quad \text{for } \text{Units} = \text{feet} \qquad \text{Bathurst (1985)}$$

$$\frac{n}{H^{1/6}} = \frac{0.3193}{5.62 \log\left(\frac{H}{d_{84}}\right) + 4.0} \quad \text{for } \text{Units} = \text{meters}$$

where R is hydraulic radius (cross-sectional area / wetted perimeter, A/P), H is average depth of flow, and d_{84} is the channel bed surface sediment diameter for which 84 percent of the material is finer.

Flow Independent Methods. Although Manning's n is depth dependent, this relationship can be rather weak in certain channel configurations (e.g., wide channels). As such, a number of flow independent grain roughness predictors have been developed for simple application. Strickler (1923) developed an empirical equation for estimating Manning's n based on the bed surface sediment diameter for which 50 percent of material is finer, d_{50} (Maynard 1991). This equation was calibrated with laboratory data and still represents one of the most commonly applied grain roughness predictors. Meyer-Peter and Muller (1948) present a seminal work in sediment transport and in the process developed a flow independent method. Wong and Parker (2006) recalibrated this classic formula using the Meyer-Peter and Muller data set and updating applied assumptions. Maynard (1991) presented a recalibration of the Strickler-type equation for a laboratory study of large, angular material often used in channel bank stabilization measures such as riprap. Though particle shapes differ from alluvial material, similarities in Maynard's equations to previous work show grain diameter is more influential to roughness than particle shape. Table presents a summary of these techniques and their calibration ranges.

$n = 0.0389d_{50}^{1/6}$	for $d_{50} = \text{feet}$	Strickler (1923)
$n = 0.0474d_{50}^{1/6}$	for $d_{50} = \text{meters}$	
$n = 0.0354d_{90}^{1/6}$	for $d_{90} = \text{feet}$	Wong and Parker (2006)
$n = 0.0431d_{90}^{1/6}$	for $d_{90} = \text{meters}$	
$n = 0.0360d_{90}^{1/6}$	for $d_{90} = \text{feet}$	Maynard (1991)
$n = 0.0439d_{90}^{1/6}$	for $d_{90} = \text{meters}$	

where d_{90} is the channel bed surface sediment diameter for which 90 percent of material is finer.

Form Roughness. Depending on height, area, and distribution and constituent sediment characteristics, the influence of alluvial bedforms on channel flow patterns can be extremely pronounced. The ability to model the influence of bedforms on hydraulic roughness adds significant complexity to roughness calculations, but consideration of these features is often critical to successfully estimate hydraulic resistance, especially in sand bed channels or where bedform height generally exceeds grain diameter. Three techniques are presented below for prediction of bedform effects on roughness: Brownlie (1981), Engelund and Hansen (1967), and van Rijn (1984). Table 2Table presents a summary of these techniques and their calibration ranges.

Brownlie (1981) presents a method for predicting effects of bedforms on flow depth with known discharge and slope. This method was derived through dimensional analysis, statistical analysis of an extensive laboratory and field database, and fundamental hydraulic principles. The technique was developed for predicting flow depth for wide sand bed channels. However, Brownlie's equations may be rearranged to predict channel roughness. Brownlie (1981) acknowledges that there is a transitional region between lower (dunes and ripples) and upper (flat bed and

antidunes) flow regimes, but for the purpose of the analysis, he suggests approximate distinguishing criteria.

Engelund (1966) and Engelund and Hansen (1967) developed a method to account for bedform effects on flow and sediment transport. This technique applies the Einstein partition differentiating between skin and form drag, which assumes that total shear stress is approximately the sum of form and skin components at constant velocity. This method was derived through dimensional analysis, statistical analysis of laboratory data of Guy et al. (1966), and fundamental hydraulic principles. This method is applicable only to relatively fine grained, dune covered beds (Vanoni 2006). Engelund and Hansen (1967) did not recommend use of this method for median grain sizes (d_{50}) of less than 0.15 mm or extremely heterogeneous sediment mixtures (Vanoni 2006).

In a thorough investigation of sediment transport, van Rijn (1984abc) presented an iterative process for estimating hydraulic roughness due to grain and form elements. Similar to Brownlie (1981), van Rijn's method was developed through a combination of dimensional analysis, statistical analysis of laboratory and field data for sand bed channels, and fundamental hydraulic principles. His analysis is more complex than that of Brownlie due to the iterative nature of the calculations.

Table 2. Notable Features of Various Grain and Form Roughness Predictors.				
Author (Year)	Data	Independent Variables	Range of Calibration	Remarks
Flow Dependent Grain Roughness Calculators				
Jarrett (1984)	Field	R, S_0	$0.5 < R < 5.51 \text{ ft}$ ($0.15 < R < 1.68 \text{ m}$), $0.002 < S < 0.04$	High-gradient mountain streams
Limerinos (1970)	Field	R, d_{84}	$1.02 < R < 10.9 \text{ ft}$ ($0.31 < R < 3.32 \text{ m}$), $0.062 < d_{84} < 2.45 \text{ ft}$ ($19 < d_{84} < 747 \text{ mm}$)	
Bathurst (1985)	Field	H, d_{84}	$0.33 < H < 5.25 \text{ ft}$ ($0.102 < H < 1.60 \text{ m}$), $0.371 < d_{84} < 2.428 \text{ ft}$ ($113 < d_{84} < 740 \text{ mm}$)	High-gradient mountain streams ($0.004 < S < 0.04$); Assumed $H=R$
Flow Independent Grain Roughness Calculators				
Strickler (1923)		d_{50}	Unknown	
Wong and Parker (2006)	Lab	d_{90}	$0.00125 < d_m < 0.094 \text{ ft}$ ($0.38 < d_m < 28.65 \text{ mm}$)	Corrected Meyer-Peter and Muller (1948) relation (d_m = median particle size)
Maynard (1991)	Lab	d_{90}	$0.015 < d_{90} < 0.440 \text{ ft}$ ($4.57 < d_{90} < 134 \text{ mm}$)	Large, angular material (riprap)
Form Roughness Calculators				
Brownlie (1983)	Lab, Field	H, S, d_{50}, σ_g	$0.082 < R < 55.8 \text{ ft}$ ($0.025 < R < 17 \text{ m}$), $2.9 \times 10^{-4} < d_{50} < 0.0092 \text{ ft}$ ($0.088 < d_{50} < 2.8 \text{ mm}$), $3.0 \times 10^{-6} < S_0 < 0.037$, $\sigma_g \leq 5$	Sand bed rivers. Only approximate flow regime delineation criteria are used in HYDROCAL.
Engelund and Hansen (1967)	Lab	H, S, d_{50}	$6.2 \times 10^{-4} < d_{50} < 0.0031 \text{ ft}$ ($0.19 < d_{50} < 0.93 \text{ mm}$), $1.3 < \sigma_g < 1.6$	Sand bed rivers. Dune bedforms only.
van Rijn (1984)	Lab, Field	H, S_0, d_{50}, d_{90}	$0.328 < R < 52.5 \text{ ft}$ ($0.10 < R < 16 \text{ m}$), $6.2 \times 10^{-4} < d_{50} < 0.0118 \text{ ft}$ ($0.19 < d_{50} < 3.6 \text{ mm}$), S_0 and d_{90} data not provided	Sand bed rivers. Iterative techniques are applied.

Vegetative Roughness. Vegetation induces large scale disturbance to the flow profile, inducing form drag on system hydraulics. Prediction of vegetative roughness is highly problematic and uncertain due to the wide array of quantitative and qualitative parameters that must be

included to accurately account for vegetative flow disturbances (Fischenich 1997). These parameters may include but are not limited to: vegetation type; plant size, shape, and rigidity; stand density; composition of mixed vegetative assemblages (e.g., riparian communities with grasses, sedges, willows and cottonwoods); and seasonality issues (e.g., summer leaf-on vs. winter leaf-off resistances). A variety of roughness calculation methods have been presented to overcome this difficulty (e.g., Baptist et al. 2007, Jarvela 2005, Kouwen et al. 1981, Lopez and Garcia 2001, Nepf 1999, Petryk and Bosmajian 1975, Wilson 2007). Although there is a significant body of literature on the subject, a knowledge gap remains in consistent, robust prediction of vegetative roughness. However, the literature abounds with observations of critical system processes. One of the most important considerations in vegetative roughness prediction is relative influence of plant height on flow depth. Emergent vegetation is herein defined as vegetation that is greater in height than the flow depth; submerged vegetation height is less than flow depth. From the immense body of vegetative roughness literature, two studies have emerged with generic applicability: Fischenich (2000) and Freeman et al. (2000).

Fischenich (2000). Fischenich (2000) presented a method of estimating roughness based purely on the theories of conservation of linear momentum and drag. For steady, uniform flow, his approach can be summarized:

$$n = \frac{k_n R^{1/6}}{\left(\frac{U}{u_*}\right)\sqrt{g}} \quad \text{where} \quad \frac{U}{u_*} = \begin{cases} \sqrt{\frac{2}{A_d C_d R}} & \text{for Emergent} \\ \frac{2.5}{H}(X + Y) & \text{for Submerged} \end{cases}$$

where h_p is vegetation height, A_d is vegetation density per unit channel length, C_d is an empirical dimensionless drag coefficient, z is distance from the bed, and:

$$\begin{aligned} X &= 1.26 h_p^2 \frac{2h_p}{11C_d A_d} \left[1 - e^{-5.5C_d A_d} \right] \\ Y &= (H - 0.95h_p) \left[\ln \left(\frac{H}{Kh_p} - \frac{0.95}{K} \right) - 1 \right] - (0.05h_p) \left[\ln \left(\frac{0.05}{K} \right) - 1 \right] \\ K &= 0.13 e^{-((C_d A_d - 0.4)^2)} \end{aligned}$$

The greatest obstacle to application of the Fischenich (2000) equations is prediction of the combined $C_d A_d$ term. Fischenich (1996) and Fischenich and Dudley (2000) discussed methods of calculating these parameters in field environments and presented calibrated values from laboratory tests.

Freeman, Rahmeyer, and Copeland (2000). Freeman et al.'s (2000) method was developed through dimensional analysis and calibrated with data from laboratory testing of live vegetation. Two equations were developed for prediction of Manning's n .

For Submerged Vegetation:

$$n = 0.183 k_n \left(\frac{E_s A_s}{\rho A_i u_*^2} \right)^{0.183} \left(\frac{h_p}{H} \right)^{0.243} (MA_i)^{0.273} \left(\frac{\nu}{Ru_*} \right)^{0.115} \left(\frac{R^{2/3} S_0^{1/2}}{u_*} \right)$$

For Emergent Vegetation:

$$n = (3.487 * 10^{-5}) k_n \left(\frac{E_s A_s}{\rho A_i^* u_*^2} \right)^{0.150} (MA_i^*)^{0.166} \left(\frac{Ru_*}{\nu} \right)^{0.622} \left(\frac{R^{2/3} S_0^{1/2}}{u_*} \right)$$

where E_s is modulus of plant stiffness $\left(E_s = \frac{F_{45} H^2}{3I} \right)$, F_{45} is horizontal force necessary to bend a

plant stem 45 degrees, I is second moment of inertia of plant stem cross-section, D_s is stem diameter, A_s is total cross-sectional area of all plant stems measured at a height of $h_p/4$ where h_p is total plant height, A_i is frontal area of the plant blocking flow ($A_i = h_p W_e$), M is plant density (plants/m²), h_p' is leaf mass height, W_e is leaf mass width, and A_i^* is effective blockage area of emergent vegetation ($A_i^* = [H - (h_p - h_p')]W_e$, Figure 1).

A significant challenge in applying this technique is the determination of the modulus of stiffness, E_s . The authors defined the modulus theoretically and present laboratory values of E_s for a variety of plant species. However, they also provided an empirical model for modulus based on plant height and diameter:

$$E_s = 1.597 * 10^5 \left(\frac{h_p}{D_s} \right) + 454 \left(\frac{h_p}{D_s} \right)^2 + 37.8 \left(\frac{h_p}{D_s} \right)^3 \quad E_s = lb_f / ft^2$$

$$E_s = 7.648 * 10^6 \left(\frac{h_p}{D_s} \right) + 2.174 * 10^4 \left(\frac{h_p}{D_s} \right)^2 + 1.809 * 10^3 \left(\frac{h_p}{D_s} \right)^3 \quad E_s = N / m^2$$

A significant strength of the Freeman et al. (2000) approach is the ability to estimate roughness in mixed vegetative assemblages by combining relevant terms into “stand averaged” vegetation properties:

$$M_{total} = \sum M_i$$

$$A_{i,avg} = \sum \left(A_i \frac{M_i}{M_{total}} \right) \quad E_{s,avg} = \sum \left(E_{s,i} \frac{M_i}{M_{total}} \right) \quad A_{s,avg} = \sum \left(A_{s,i} \frac{M_i}{M_{total}} \right)$$

$$h_{p,avg} = \sum \left(h_{p,i} \frac{M_i}{M_{total}} \right) \quad h_{p',avg} = \sum \left(h_{p',i} \frac{M_i}{M_{total}} \right) \quad A_{i^*,avg} = \sum \left(A_{i^*,i} \frac{M_i}{M_{total}} \right)$$

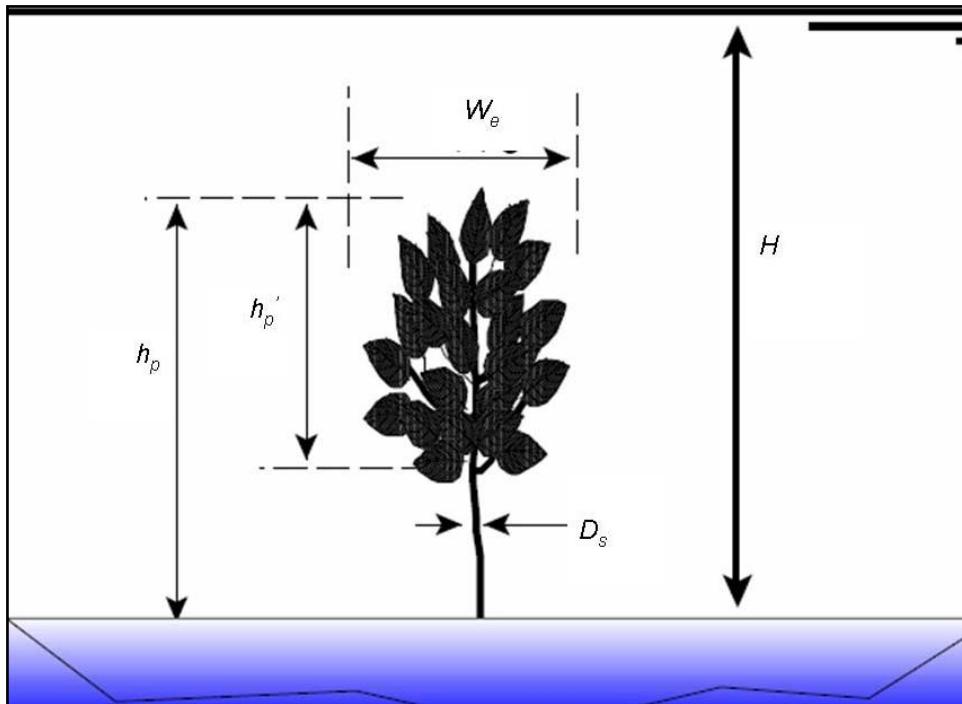


Figure 1. Schematic of idealized plant dimensions (after Freeman et al. 2000).

Composite Roughness. Thus far, hydraulic roughness has been addressed assuming a relatively homogeneous section. However, many channels, particularly under flood conditions, exhibit varying roughness conditions throughout the lateral domain (e.g., vegetated floodplain of a sand bed stream). In these channels a composite or cross-section averaged roughness, n_c , must be calculated to most accurately represent variable cross-sectional roughness contributions. For a sample channel and floodplain cross-section with seven approximately homogenous subsections (e.g., Figure 2), Manning's n may be estimated for each subsection and the composite roughness determined through various averaging approaches. Yen (2002) provides 17 such equations based on varying theoretical assumptions (Table 3 presents 12 such equations and the theoretical basis for each).

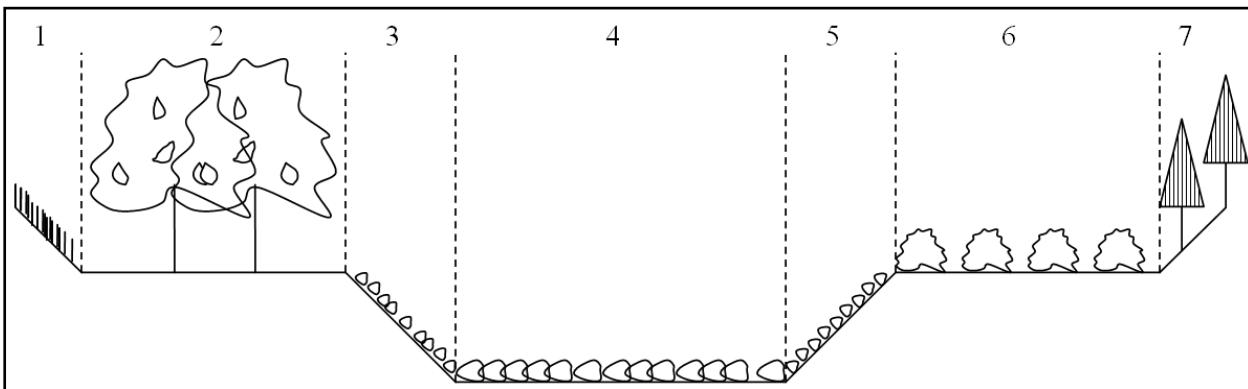


Figure 2. Typical cross-section requiring composite roughness estimates.

Table 3. Equations for Compositing Roughness.

Theoretical Basis	Source		
	Equation for n_c		
Total discharge is the sum of subsection discharge	Yen (2002)	Felkel (1960)	Lotter (1933)
	$\frac{A}{\sum A_i}$	$\frac{P}{\sum P_i}$	$\frac{PR^{5/3}}{\sum \frac{PR_i^{5/3}}{n_i}}$
Total resistance force is the sum of subsection resistance forces	Yen (2002)	Pavlovski (1931)	Yen (2002)
	$\sqrt{\frac{1}{A} \sum (n_i^2 A_i)}$	$\sqrt{\frac{1}{P} \sum (n_i^2 P_i)}$	$\sqrt{\frac{1}{PR^{2/3}} \sum (n_i^2 PR_i^{2/3})}$
Total shear velocity is weighted sum of subsection shear velocity	Los Angeles District Method	Yen (1991)	Yen (1991)
	$\frac{\sum n_i A_i}{A}$	$\frac{\sum n_i P_i}{P}$	$\frac{\sum n_i PR_i^{1/3}}{PR^{1/3}}$

HYDraulic ROughness CALculator (HYDROCAL). Accurate quantification of resistance in streams and rivers can be challenging. Due to this complexity, choice of resistance coefficients often centers on professional judgment rather than exact science. The importance of accurately quantifying this parameter encourages use of multiple estimates of n from a variety of methods. This “weight-of-evidence” approach to predicting Manning’s n creates a more robust estimate and can provide significant reduction in uncertainty, provided methods are chosen that are tailored or recommended for particular settings.

To simplify the process, techniques summarized above have been condensed into a single tool for predicting roughness. This HYDraulic ROughness CALculator (HYDROCAL) allows a user to calculate roughness over a dozen different ways, provides graphical outputs for easy comparison of methods, and calculates composite roughness. This section provides an overview of HYDROCAL and guidance for application. Two notable limitations were applied to simplify model development: 1) HYDROCAL considers roughness prediction in the form of Manning (n) only, and 2) HYDROCAL only applies SI units.

To provide an easily transferable, user-friendly computational tool, HYDROCAL was designed using Microsoft Excel, a spreadsheet software format commonly in use by many agencies and practitioners. The model was constructed using a set of pre-programmed forms, buttons, and equations to create an interactive graphical user interface. Three notable instructions are:

1. When opening the model in Microsoft Excel, always “Enable Macros” to ensure that the model is fully functional. If the model will not open properly, verify that security settings (Tools/Options/Security/Macro Security) are medium or low.
2. To apply a prediction method, select the “Check Box” next to the technique (e.g. select Fischenich (2000) to apply Fischenich’s vegetative predictor).
3. Cells are color coded to indicate the type of value within (Figure 3).

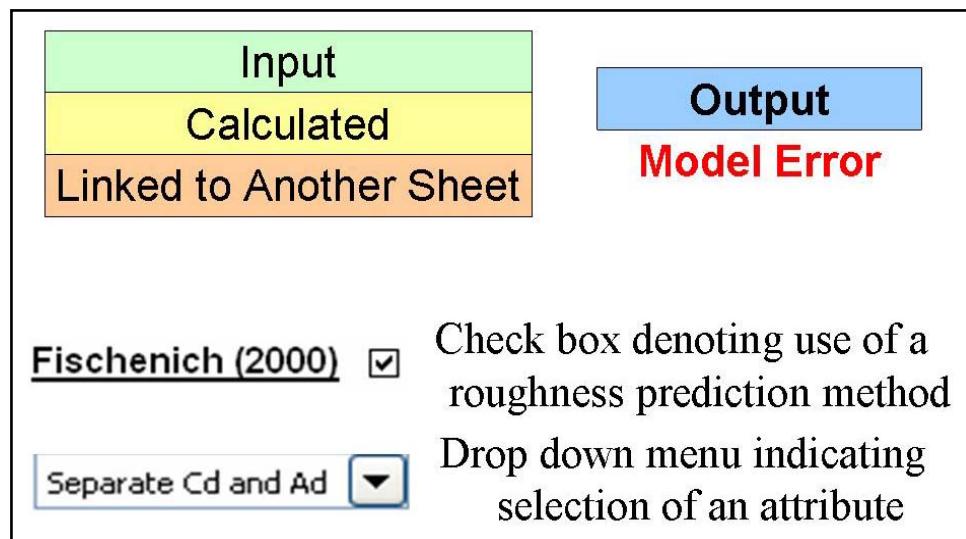


Figure 3. Graphical User Interface (GUI) Legend.

HYDROCAL contains three basic computational modules: 1) “RoughnessEstimator”, 2) “VegRoughnessEstimator”, and 3) “CompositeCalcs”. These modules estimate roughness in an individual subsection from a variety of factors (1, 2) and composite those estimates into cross-section averaged estimates (3).

“RoughnessEstimator.” The “RoughnessEstimator” module provides users with the ability to calculate and compare non-vegetative roughness estimates for a given channel subsection. For the purposes of HYDROCAL, many analytical techniques were applied assuming wide channel geometry ($H \approx R$) and uniform flow ($S_f \approx S_w \approx S_0$). Additional caveats, assumptions, and notable features of the techniques presented are:

Total Roughness Predictors

- Direct Measurement: Input hydraulic radius, slope, and velocity
- Best Professional Judgment: Apply professional judgment in field to estimate roughness (estimates from multiple experienced river scientists are recommended)
- Photographic Comparison: Input roughness value from corresponding photograph. Hyperlinks are provided in the model to references.
- Cowan’s Method: Hyperlinks to related documentation are provided for estimation of resistance components.
- Chow (1959) Tables:
 - Select the “Chow” worksheet and check the box next to the channel type that most closely resembles the channel in question
 - Return to the “RoughnessEstimator” worksheet and choose an appropriate estimate of roughness for the range of values presented.
 - An error message will appear if more than one box is selected.

Grain Roughness Predictors

- Choose desired predictors and input required parameters
- Refer to this document for calibration ranges when selecting methods.

Form Roughness Predictors

- Constants assumed for all form roughness predictions:
 - $R_g = 1.65$, $v = 1.005 * 10^{-6} \text{ m}^2/\text{s}$, $\rho = 1000 \text{ kg/m}^3$
- Choose desired predictor(s) and input required parameters
- Refer to this document for calibration ranges when selecting methods.
- Brownlie (1981):
 - Transitional region between lower and upper regime was ignored and approximate delineation criteria identified by Brownlie (1981) were used.
 - The logarithm of grain size distribution was assumed Gaussian.
- Engelund and Hansen (1967)
 - Einstein partition was assumed.
 - Dunes only
- van Rijn (1984)
 - Shields' critical mobility parameter, $\tau_{*,cr}$, was assumed to be defined by the relations of van Rijn (1984a)
 - After inputting parameters, click the button provided to iterate velocity to obtain the correct solution. If iteration will not converge, an error message will appear.

“VegRoughnessEstimator.” Vegetative elements often induce roughness of a different character and magnitude compared with other forms of roughness; therefore, vegetative roughness was isolated as a separate roughness prediction module, “VegRoughnessEstimator”. Format and application of this module are analogous to “RoughnessEstimator,” though inputs differ due to differing flow character through vegetation. Additionally, wide channel geometry ($H \approx R$) and uniform flow ($S_f \approx S_w \approx S_0$) were assumed in this module as well. Submergence is determined by HYDROCAL. Additional caveats, assumptions, and notable features of techniques presented are:

- Fischenich (2000): Estimates of drag and area coefficients (C_d , A_d) are needed for application of this technique. Fischenich (1996) and Fischenich and Dudley (2000) provide guidance on assessing these values in field settings. In HYDROCAL these values may be input as combined coefficients (C_dA_d) or separate values (C_d , A_d). Recommended coefficient values are provided in HYDROCAL.
- Freeman, Rahmeyer, and Copeland (2000)
 - A water temperature of 20°C is assumed for all calculations.
 - A maximum of five plants may be entered. If fewer than five plants are desired, leave input cell values blank.
 - All plant dimensions should be input based on Figure 1.
 - Modulus of plant stiffness, E_s , is calculated by equations presented in this document for all plant species.

Output of Subsection Calculators. Roughness estimates from “RoughnessEstimator” and “VegRoughnessEstimator” are summarized in the “Output” worksheet. This worksheet creates an environment for easy visual and tabular comparison of roughness from various subsection resistance predictions. This module also allows for calculation of Darcy-Weisbach and Chezy coefficients based on predictions and an estimate of hydraulic radius.

“CompositeCalcs.” As previously discussed, roughness prediction techniques often apply to only a subsection of a channel (e.g., the vegetated floodplain or active channel) though cross-section averaged estimates are desired. The “CompositeCalcs” module presents 12 techniques for compositing roughness. This module presents estimates of subsection roughness obtained in “RoughnessEstimator” and VegRoughnessEstimator” modules and allows the user to enter a Manning’s n and channel geometry parameters (A, P) for as many as ten subsections. Cross-section averaged values of Manning’s n are presented in both tabular and graphical formats for easy comparison.

APPLICATION: HAM BRANCH. To demonstrate the utility of HYDROCAL, the model is applied to Ham Branch, a tributary to the Trinity River in Fort Worth, Texas. Ham Branch is a small urban stream with a gravel channel and vegetated floodplains (Figure 4). Estimates of Manning’s n were required as input for further hydraulic analyses with HEC-RAS. HYDROCAL was applied to compare different estimates of resistance in the floodplains and main channel (Figure 5). The “CompositeCalcs” module was then applied to estimate cross-section averaged resistance from these subsection estimates (Figure 6).

As demonstrated, Manning’s n varies significantly between predictive techniques. This highlights the need to examine multiple estimates and apply professional judgment to arrive at the most likely estimate. Moreover, the range of uncertainty in a given estimate is quantified based on the range predicted values and sensitivity analyses can be conducted to assess the potential range of outcomes.

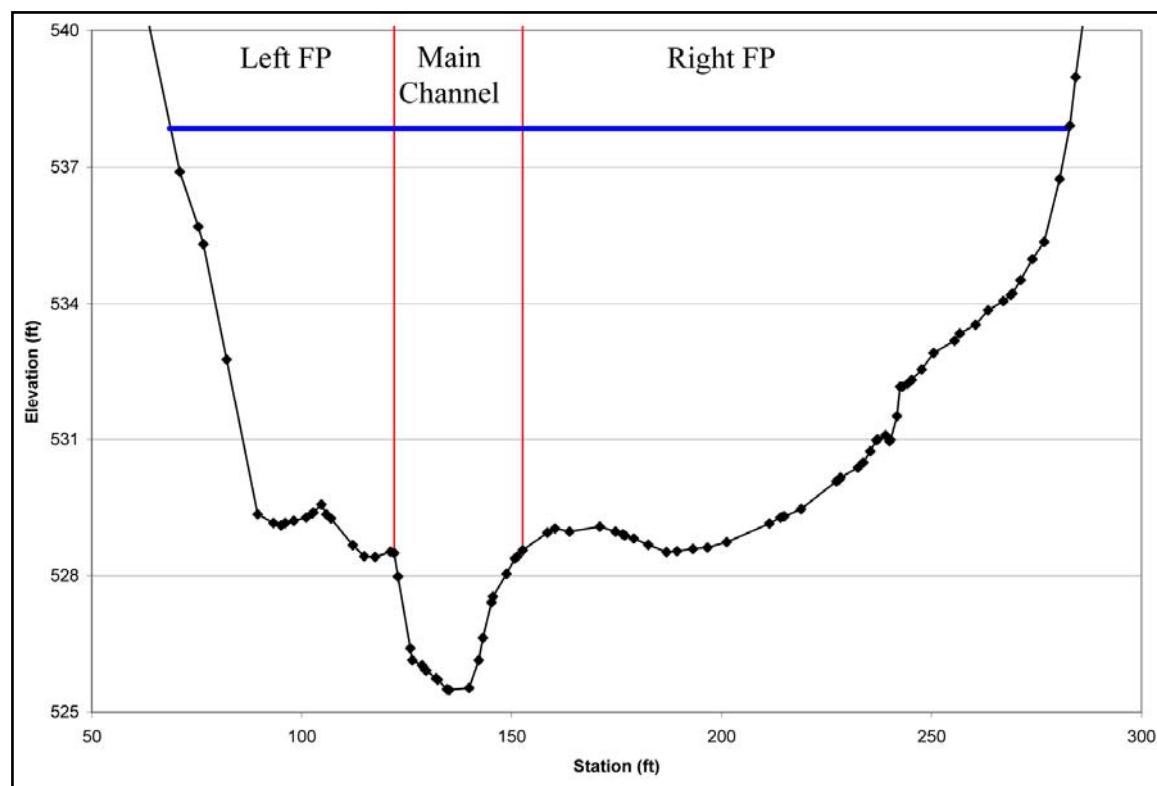


Figure 4. Sample cross-section of Ham Branch with subsections identified.

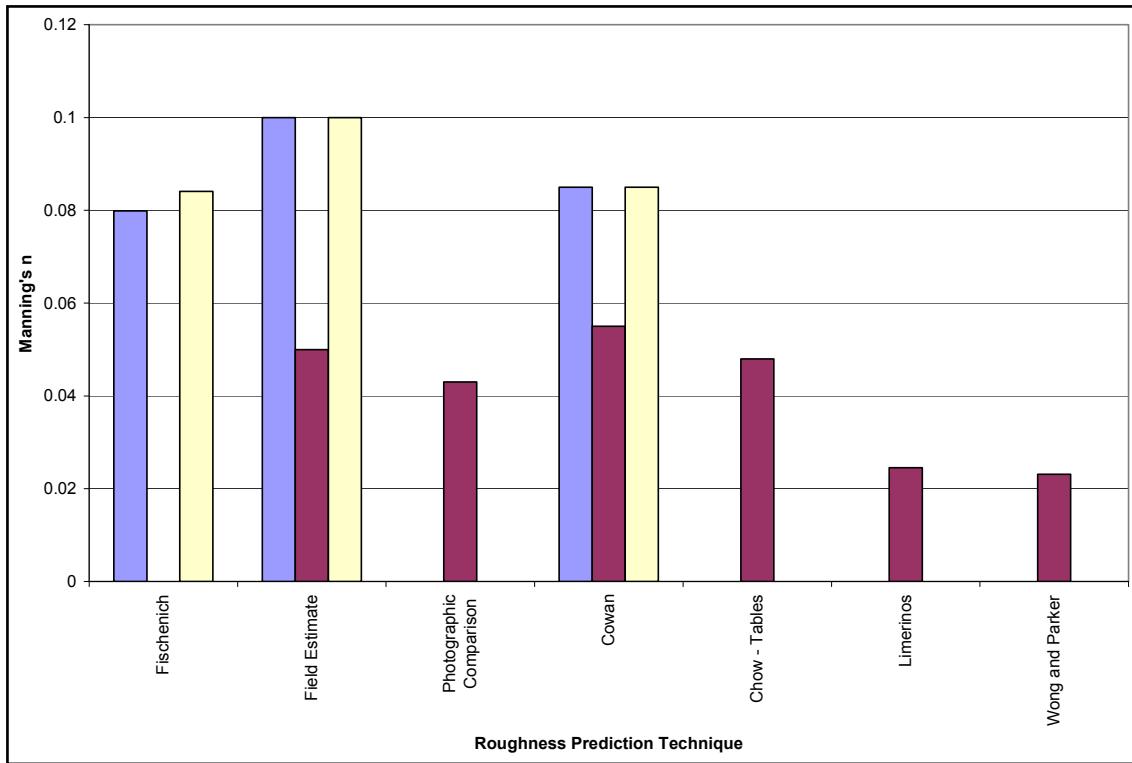


Figure 5. HYDROCAL subsection roughness estimates for the left floodplain (blue), main channel (red), and right floodplain (white).

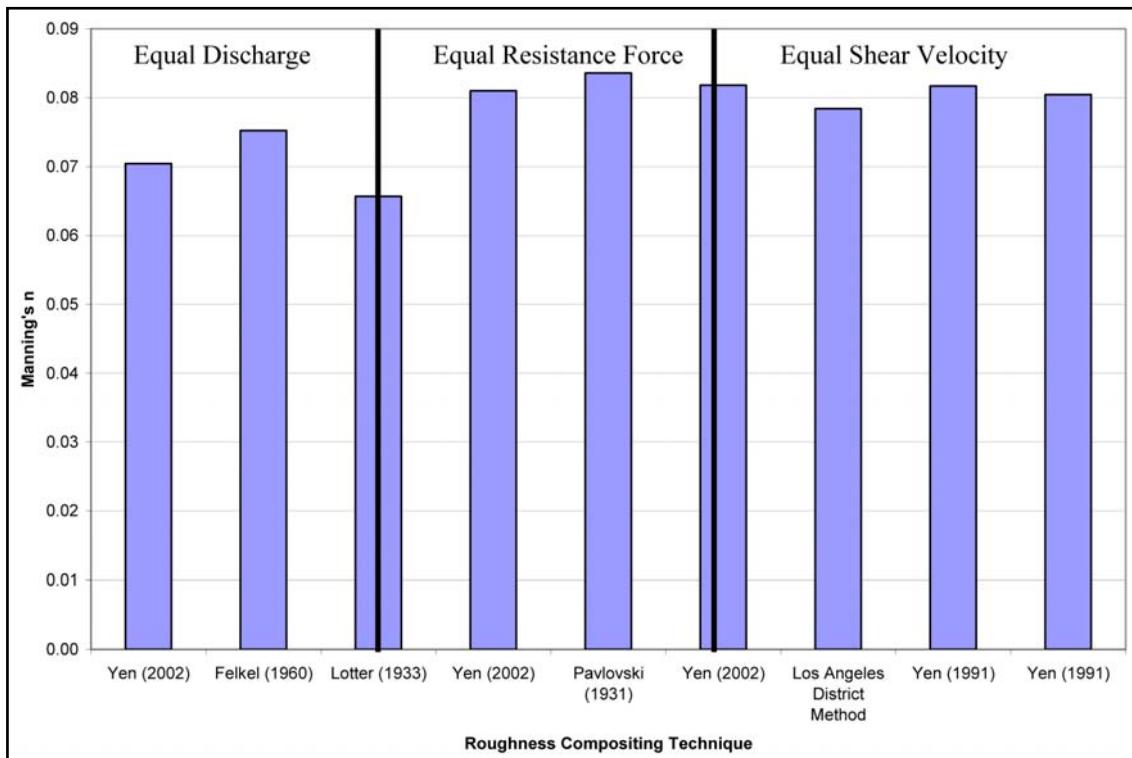


Figure 6. HYDROCAL composite roughness estimates.

SUMMARY: This document has presented a summary of hydraulic roughness prediction techniques and their theoretical bases and summarized those techniques in the form of a model for predicting hydraulic resistance. This HYDraulic ROughness CALculator (HYDROCAL) was applied to Ham Branch to demonstrate its utility for estimating total, grain, bedform, and vegetative roughness coefficients and compositing those estimates into a cross-sectional averaged roughness value. The diversity of methodologies allows users to create a “weight-of-evidence” approach for estimating hydraulic roughness and provides users with the potential range of uncertainty. Practitioners should note that professional judgment should always be applied when selecting a roughness coefficient or estimation method due to the importance of this value in hydraulic design calculations. HYDROCAL merely calculates a range of potential resistance values from various techniques; the model does *not* provide a final estimate of hydraulic roughness and care should be taken in selecting this value.

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